Dipole-based description of pp interaction

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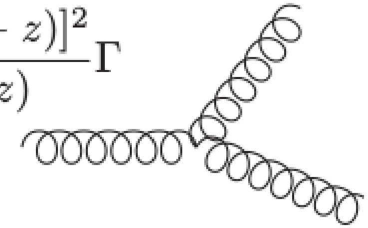
Fast moving proton

- Static hadron is complicated
- In high momentum frame of reference highly virtual fluctuations get energy → become quasi-real
- Partonic degrees of freedom – an appropriate description of a proton at high energy
- Convenient way of the description of such phenomena as multiplication interactions and saturation is use of transverse space coordinates

Color dipoles

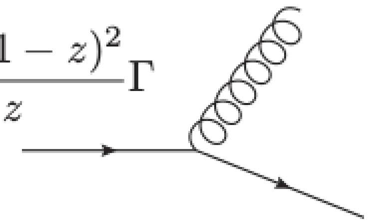
- Gluon emission dominates

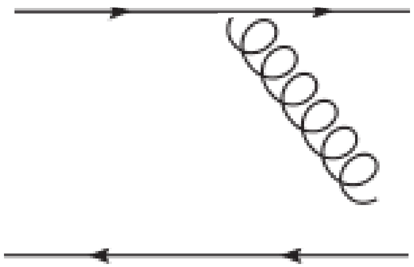
$$2N_c \frac{[1 - z(1 - z)]^2}{z(1 - z)} \Gamma$$



- In a large-N_c limit a gluon emission is equivalent to the splitting of a color dipole:

$$(N_c^2 - 1)/N_c \frac{1 + (1 - z)^2}{z} \Gamma$$





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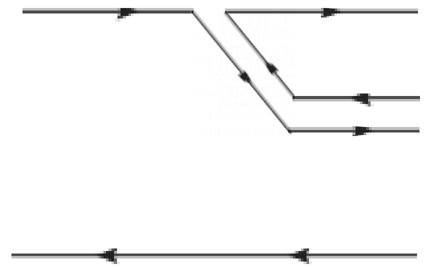
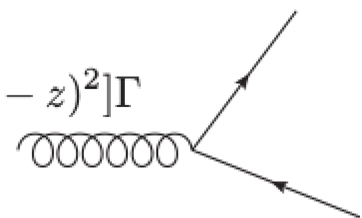


$$N_f [z^2 + (1 - z)^2] \Gamma$$



- Evolution described by the BFKL [1-2]

[1] A.H. Mueller and B. Patel, Nucl. Phys. B425 (1994) 471 [hep-ph/9403256].
[2] A.H. Mueller, Nucl. Phys. B437 (1995) 107 [hep-ph/9408245].

pp collisions as an interaction of dipoles

- A proton is considered as a set of dipoles in transverse plane

- A dipoles ends are distributed according to the Gaussian distribution:

$$\rho(x,y) = \frac{1}{2\pi {r_0}^2} e^{-\frac{r^2}{r_0^2}}$$

with parameter r_0 related to the proton radius:

$$r_0 = \sqrt{\frac{2}{3}} \sqrt{< r_N^2 >}$$

- A probability amplitude of the elementary collision is given according to Muller model prescription[3-5]:

$$f = \frac{\alpha_S^2}{8} \ln^2 \frac{(\vec{r}_1 - \vec{r}_3)^2 (\vec{r}_2 - \vec{r}_4)^2}{(\vec{r}_1 - \vec{r}_4)^2 (\vec{r}_2 - \vec{r}_3)^2}$$

[3] G. P. Salam Nucl. Phys. B461 (1996) 512–538, hep-ph/9509353.
[4] A.H. Mueller, Nucl. Phys. B437 (1995) 107 [hep-ph/9408245].
[5] E. Avsar, G. Gustafson, and L. Lönnblad, JHEP 07 (2005) 062 [hep-ph/0503181].

pp collisions as an interaction of dipoles

- For multiple interactions an eikonal approach has been used

- The probability of elementary interaction is given by:

$$p_{ij} = 1 - e^{-f_{ij}}$$

- The total probability of the interaction of two protons:

$$p = 1 - e^{-\sum\limits_{i,j} f_{ij}}$$

- The proton-proton collision profile function is obtained by averaging at fixed impact parameter:

$$\sigma(b) = < 1 - e^{-f} >$$

Calculation of $\sigma(b)$ at large impact parameters

- Take one dipole for both target B and projectile A with relative impact parameter $\vec{b} = \vec{r}_A - \vec{r}_B$

- Relative coordinates:

$$\begin{aligned} \vec{r}_1 &= \vec{r}_A + \vec{\delta}_1, \\ \vec{r}_2 &= \vec{r}_A + \vec{\delta}_2, \\ \vec{r}_3 &= \vec{r}_B + \vec{\delta}_3, \\ \vec{r}_4 &= \vec{r}_B + \vec{\delta}_4. \end{aligned}$$

- Relative coordinates are distributed according to Gaussian distribution:

$$\rho(\vec{\delta}_j) = \frac{1}{2\pi {r_0}^2} e^{-\frac{\vec{\delta}_j^2}{{r_0}^2}}$$

Calculation of $\sigma(b)$ at large impact parameters

- at $b \gg r_0$ we can expand the logarithm:

$$\ln((\vec{r}_1 - \vec{r}_3)^2) \simeq \ln \ \vec{b}^2 + 2\frac{\vec{b}(\vec{\delta}_1 - \vec{\delta}_3)}{b^2} + \frac{(\vec{\delta}_1 - \vec{\delta}_3)^2}{b^2} - 2\frac{(\vec{b}(\vec{\delta}_1 - \vec{\delta}_3))^2}{b^4}$$

and obtain f:

$$f \simeq \frac{\alpha_S^2}{8}\Big[\frac{1}{b^2}\Big((\vec{\delta}_1 - \vec{\delta}_3)^2 + (\vec{\delta}_2 - \vec{\delta}_4)^2 - (\vec{\delta}_1 - \vec{\delta}_4)^2 - (\vec{\delta}_2 - \vec{\delta}_3)^2\Big) -$$
$$-\frac{2}{b^4}\Big(((\vec{\delta}_1 - \vec{\delta}_3)\vec{b})^2 + ((\vec{\delta}_2 - \vec{\delta}_4)\vec{b})^2 - ((\vec{\delta}_1 - \vec{\delta}_4)\vec{b})^2 - ((\vec{\delta}_2 - \vec{\delta}_3)\vec{b})^2\Big)\Big]^2$$

- To the lowest order $< 1 - e^{-f} > \simeq < f >$

- We used Wick's theorem for the averaging of products:

$$< A_1 A_2 A_3 A_4 > = < A_1 A_2 >< A_3 A_4 > + < A_1 A_3 >< A_2 A_4 > + < A_1 A_4 >< A_2 A_3 >$$

Calculation of $\sigma(b)$ at large impact parameters

- We have

$$< \vec{\delta}_j >= 0, \; < \vec{\delta}_j^2 >= {r_0}^2, \; < {\delta_{j_x}}^2 >=< {\delta_{j_y}}^2 >= \frac{{r_0}^2}{2}$$

- Using Wick's theorem

$$< {\delta_{j_x}}^4 >= 3 \cdot \left(\frac{{r_0}^2}{2} \right)^2 = \frac{3}{4} {r_0}^4$$

- Some more examples:

$$< (\vec{\delta}_1 \vec{\delta}_3)(\vec{\delta}_1 \vec{\delta}_3) >=< (\delta_{1x}\delta_{3x} + \delta_{1y}\delta_{3y})(\delta_{1x}\delta_{3x} + \delta_{1y}\delta_{3y}) >= \frac{r_0^2}{4} + \frac{r_0^2}{4} = \frac{r_0^2}{2}$$

$$< (\vec{\delta}_1 - \vec{\delta}_3)^4 >=< (\delta_1^2 - 2\vec{\delta}_1\vec{\delta}_3 + \delta_3^2)^2 >= 2 \cdot 2r_0^4 + r_0^4 + 4\frac{r_0^4}{2} + r_0^4 = 8r_0^4$$

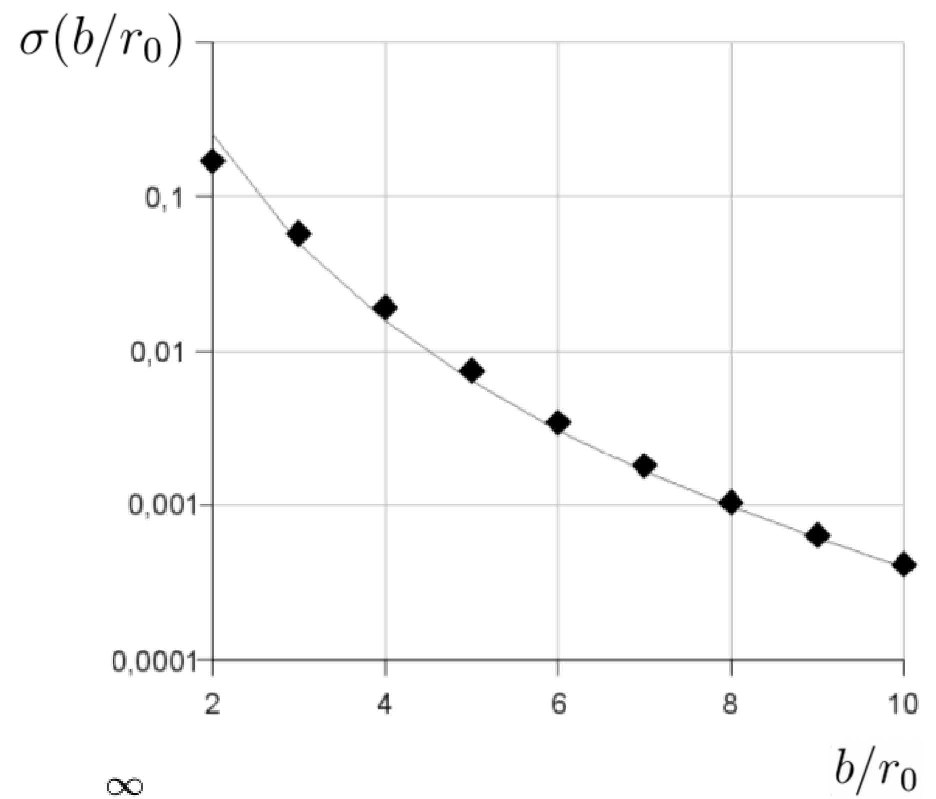
$$< (\vec{\delta}_1 - \vec{\delta}_3)^2 (\vec{\delta}_2 - \vec{\delta}_4)^2 >= 4r_0^4$$

$$< (\vec{\delta}_1 - \vec{\delta}_3)^2 (\vec{\delta}_1 - \vec{\delta}_4)^2 >= 5r_0^4$$

$\sigma(b)$ at large impact parameters

- Finally, we obtain $\sigma(b) \simeq \alpha_S^2 \left(\frac{r_0}{b}\right)^4 \simeq 1 - e^{-\alpha_S^2 \left(\frac{r_0}{b}\right)^4}$

- Comparison between asymptotics and numerical calculations

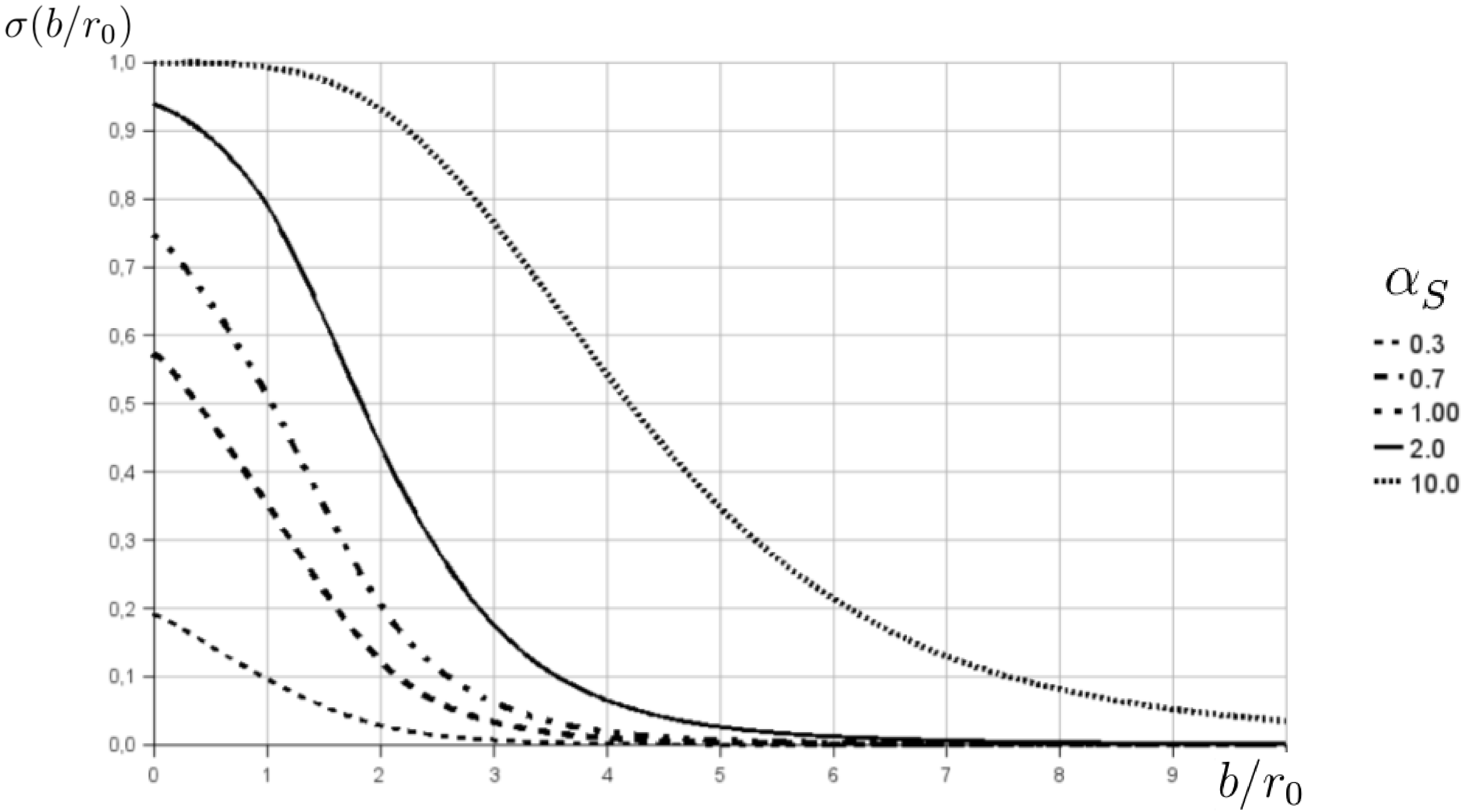


- Very rough estimation of total inelastic cross section

$$\sigma \simeq \int_0^\infty \left(1 - e^{-\alpha_S^2 \left(\frac{r_0}{b}\right)^4}\right) \, 2\pi b \, db = 2\pi \alpha_s r_0^2 \int_0^\infty t(1 - e^{-\frac{1}{t^4}}) dt \approx 5,57 \cdot \alpha_S r_0^2$$

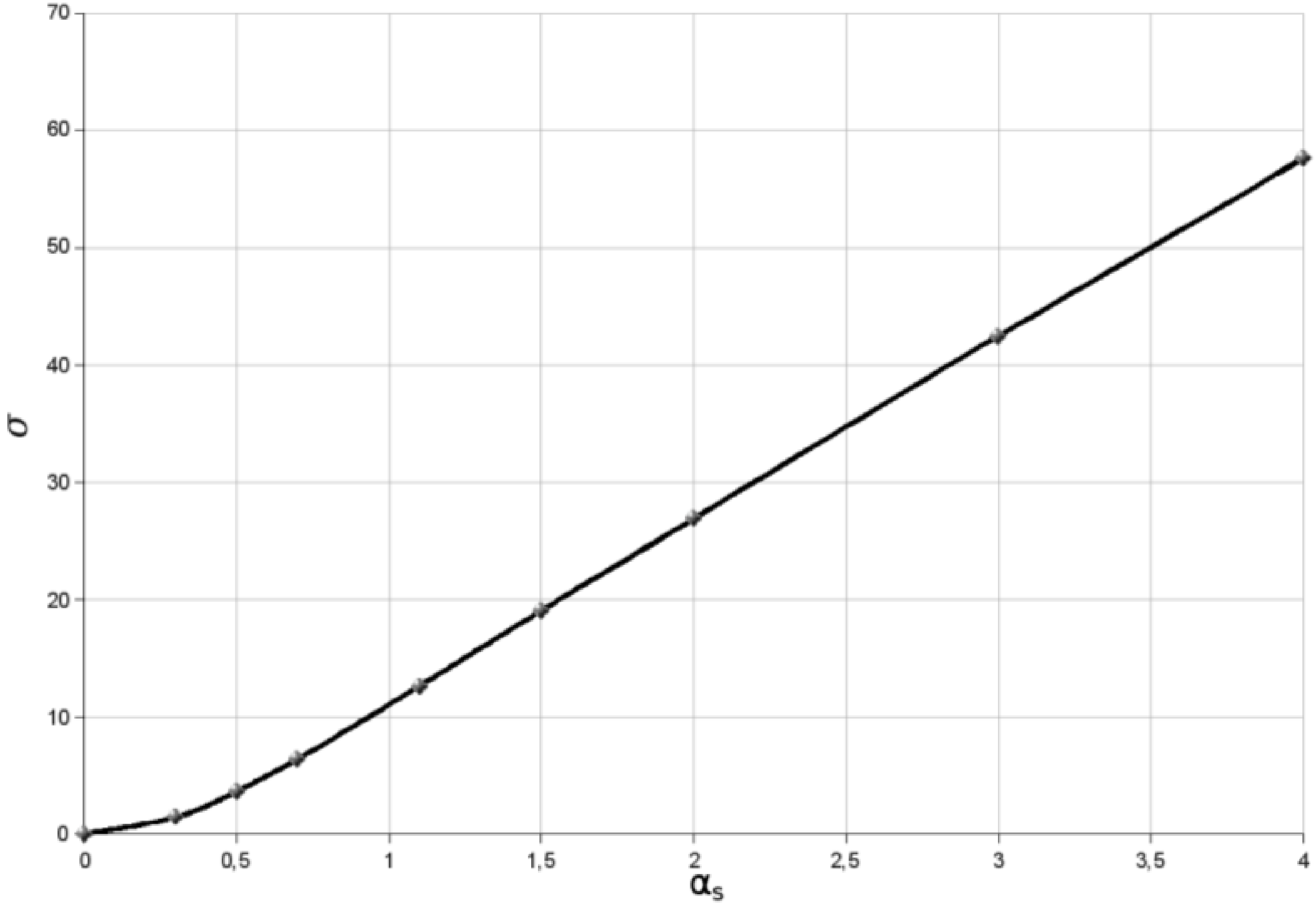
$\sigma(b)$ – numerical results

- $\sigma(b)$ at different values of α_S



Numerical results

- Total inelastic cross section as a function of α_S



Elastic differential cross section and inelastic $\sigma(b)$

- Elastic and total cross section

$$\sigma_{el} = \int \frac{d^2 q}{(2\pi)^2} |M(q)|^2 \qquad \sigma_{tot} = 2\,\mathrm{Im}\, M_{el}(q=0)$$

- Using Fourier transformation

$$t(b) = \frac{1}{(2\pi)^2} \int d^2 q e^{-i\vec{q}\vec{b}} M(q)$$

- Assuming amplitude to be purely imaginary

$$t(b) = i g(b), \quad g(b) \in \mathbb{R}$$

- We find $\sigma_{inel} = \sigma_{tot} - \sigma_{el} = \int (2g(b) - g^2(b)) d^2 b = \int \sigma(b) d^2 b.$

- So, unitarity conditions relates elastic amplitude and and inelastic collisional profile of proton [6-7]

$$g(b) = 1 - \sqrt{1 - \sigma(b)}$$

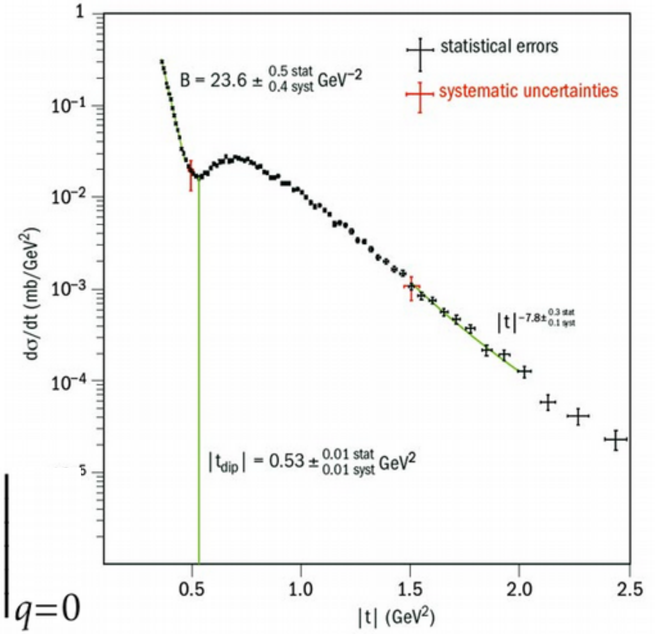
[6] M. Rybczynski, Z. Wlodarczyk, J.Phys. G 41 (2013) 015106.
[7] V. A. Schegelsky and M. G. Ryskin, Phys. Rev. D 85, 094024 (2013).

Slope of the diffraction cone

- At small t

$$\frac{d\sigma}{dt} \sim C e^{Bt}$$

where $t = -q^2 < 0,\ -t$ – momentum transfer squared



- This enables calculation of B:

$$B = \frac{1}{|M(0)|^2}\left(-\frac{1}{2q}\right)\frac{d}{dq}|M(q)|^2 \Bigg|_{q=0}$$

- Finally, we obtain

$$B = \frac{1}{2}\frac{\int b^3 g(b)db}{\int bg(b)db}$$

or

$$B = \frac{1}{2}\frac{\int b^3(1-\sqrt{1-\sigma(b)})db}{\int b(1-\sqrt{1-\sigma(b)})db}.$$

So, for dipole model $\sigma(b) \simeq 1 - e^{-\alpha_S^2\left(\frac{r_0}{b}\right)^4}$ diffraction cone slope logarithmically diverges

Taking into account confinement in dipole approach

- An amplitude

$$f = \frac{\alpha_S^2}{8} \ln^2 \frac{(\vec{r}_1 - \vec{r}_3)^2 (\vec{r}_2 - \vec{r}_4)^2}{(\vec{r}_1 - \vec{r}_4)^2 (\vec{r}_2 - \vec{r}_3)^2}$$

can be rewritten in the following form:

$$f(\vec{r}_1, \vec{r}_2 | \vec{r}_3, \vec{r}_4) = \frac{g^4}{8} [\Delta(\vec{r}_1 - \vec{r}_3) - \Delta(\vec{r}_1 - \vec{r}_4) - \Delta(\vec{r}_2 - \vec{r}_3) + \Delta(\vec{r}_2 - \vec{r}_4)]^2$$

where

$$\Delta(\vec{r}) = \int \frac{d^2\vec{k}}{(2\pi)^2} \frac{e^{i\vec{k}\cdot\vec{r}}}{k^2}$$

- Substitute $\frac{1}{k^2} \rightarrow \frac{1}{k^2 + M^2}$, where $M = 1/r_{max}$ – confinement scale [8-9]

- One obtains:

$$f = \frac{\alpha_s^2}{2} \left[K_0\left(\frac{|\vec{r}_1 - \vec{r}_3|}{r_{max}}\right) + K_0\left(\frac{|\vec{r}_2 - \vec{r}_4|}{r_{max}}\right) - K_0\left(\frac{|\vec{r}_1 - \vec{r}_4|}{r_{max}}\right) - K_0\left(\frac{|\vec{r}_2 - \vec{r}_3|}{r_{max}}\right) \right]^2,$$

where K_0 – modified Bessel function of the second kind

[8] E. Avsar, G. Gustafson, and L. Lönnblad, JHEP 01 (2007) 012 [hep-ph/0610157].
[9] C. Flensburg, G. Gustafson, and L. Lönnblad, Eur. Phys. J. C60 (2009) 233, [arXiv:0807.0325].

Taking into account confinement in dipole approach

$$f = \frac{\alpha_s^2}{2}\left[K_0\left(\frac{|\vec{r}_1-\vec{r}_3|}{r_{max}}\right)+K_0\left(\frac{|\vec{r}_2-\vec{r}_4|}{r_{max}}\right)-K_0\left(\frac{|\vec{r}_1-\vec{r}_4|}{r_{max}}\right)-K_0\left(\frac{|\vec{r}_2-\vec{r}_3|}{r_{max}}\right)\right]^2$$

- At $r \ll r_{max}$ $K_0(r/r_{max}) \simeq -\ln(r/(2r_{max}))$, that brings back to the initial formula
- At $r \gg r_{max}$ $K_0\left(\frac{r}{r_{max}}\right) \simeq \sqrt{\frac{\pi r_{max}}{2r}}e^{-\frac{r}{r_{max}}}$, that ensures exponential decrease of the $\sigma(b)$.
- Collision profile function $\sigma(b) \sim \left(\frac{r_{max}}{b}\right)^5 e^{\left(\frac{-2b}{r_{max}}\right)}$

provides finite values of the diffraction cone slope

Diffraction cone slope as a function of $\sqrt{s}$

- Dipole model

(parameters turning was performed as in [10])

- Black disk $\quad \sigma(b) = \theta(R - b)$

- Grey disk $\quad \sigma(b) = g\,\theta\left(\frac{R}{\sqrt{g}} - b\right)$

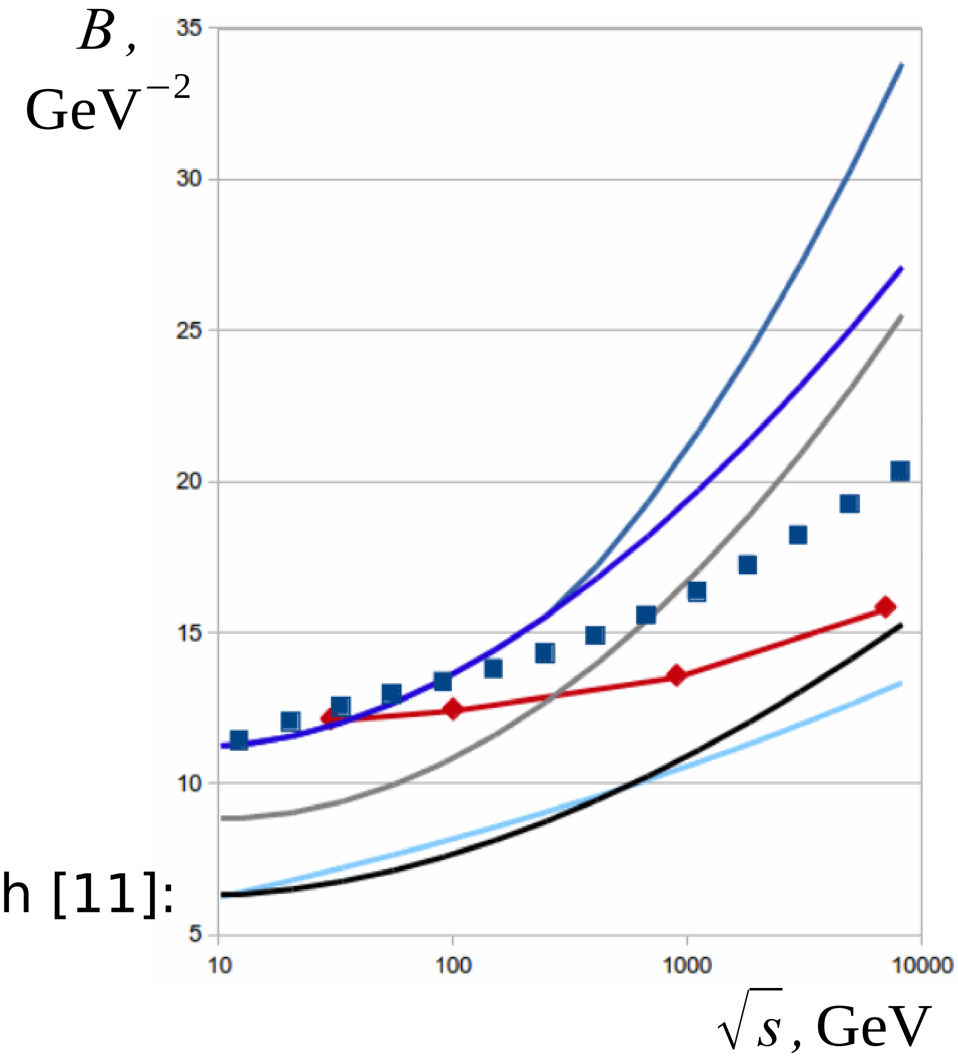
- Black Gauss $\quad \sigma(b) = e^{-\frac{b^2}{R^2}}$

- Grey Gauss $\quad \sigma(b) = g\,e^{-g\frac{b^2}{R^2}}$

- Quasi-eikonal Regge approach [11]:

$$\sigma(b) = 1 - \exp\left(-N_0\,e^{-\frac{b^2}{2\alpha^2}}\right)$$

- Experimental data fit [7]



[7] V. A. Schegelsky and M. G. Ryskin, Phys. Rev. D 85, 094024 (2013).
[10]V. Kovalenko, PoS QFTHEP2013 (2013) 052
[11] V. Vechernin, I. Lakomov, PoS (Baldin ISHEPP XXI) 072 (2012).

Conclusions

- Inelastic pp collisions at high energy were considered in transverse plane in the dipole approach

- Without accounting of confinement a power-like behavior for the collision probability is obtained

- Inclusion of the confinement ensures exponential decrease of the collision amplitude and finite slope

- The experimental data on the diffraction cone slope provide useful input for parameter turning of the model

The End

- Thank you!